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ABSTRACT

In this presentation we will investigate the mathematics and physics of a free-falling object, not for the purpose of research to discover some profound physical phenomena, but rather to enhance students' ability to gather, organize, and analyze data using state of the art hand held technology. We will use the TI-83 plus (a sophisticated graphing calculator) and the TI-CBR (a Calculator Based sonic Ranging device) along with an adequate supply of 8 to 12 cup, no special brand, coffee filters. Since the coffee filters are not consumed, a single package is generally sufficient to supply the needs of more than one class. We will use the CBR and TI-83 Plus to collect data about the coffee filter drops. We will manually analyze some of the data and then use a previously written program to expedite the task. From the information obtained during the presentation we should be able to draw a reasonable conclusion about the terminal velocity that we challenge you to verify with your students. (Author)

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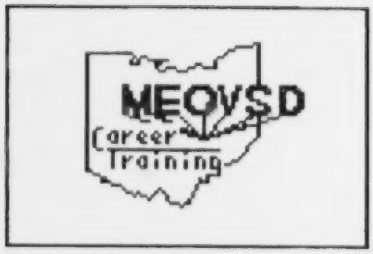

COFFEE FILTER TERMINAL VELOCITY

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COFFEE FILTER TERMINAL VELOCITY

INTRODUCTION/OVERVIEW

In this presentation we will investigate the math and physics of a free-falling object, not for the purpose of research to discover some profound physical phenomena, but rather to enhance students' ability to gather, organize, and analyze data using state of the art hand held technology.

We will use the TI-83 plus (a sophisticated graphing calculator) and the TI-CBR (a Calculator Based sonic Ranging device) along with an adequate supply of 8 to 12 cup, no special brand, coffee filters. Since the coffee filters are not consumed, a single package is generally sufficient to supply the needs of more than one class.

We will use the CBR and TI_83 Plus to collect data about the coffee filter drops. We will manually analyze some of the data and then use a previously written program to expedite the task. From the information obtained during the presentation we should be able to draw a reasonable conclusion about the terminal velocity that we,challenge you to verify with your students.

$$d = \tfrac{1}{2}at^2 + v_o t + d_o$$

if $v_o = 0$ initial velocity and $d_o = 0$ initial displacement

then $d = \tfrac{1}{2}at^2$

Displacement equals one half times acceleration times time squared. Where acceleration is the result of a net force different from zero.

When an object moves through a fluid there is resistance to that movement related to the size of the object projected in the direction of motion (cross-sectional area), the nature of the surface (drag coefficient), the viscosity of the fluid and the speed of the object. This resistance is called Drag Force.

An object free falling in a fluid is subject to acceleration due to the force of gravity as reduced by drag force. When the drag force, which increases with speed, equals the weight of the object no further acceleration occurs and the object is said to be at "Terminal Velocity"

$$F_{termnldrag} = mg \quad m = \text{mass}, \quad g = 9.8\ \text{Nm/sec}^2$$

For a small object falling through a high viscosity fluid the drag force is directly proportional to the object's velocity.

$F_{drag} = kv$ When $mg = kv$ then $v_{termnl} = mg / k$

For heavier larger cross-sectional area objects free falling in air the drag force is directly proportional to the square of the object's velocity.

$$F_{drag} = Pv^2$$

Where P is proportional to area, drag coefficient and viscosity.

When $F_{drag} = mg = Pv^2$ then $v^2 = \frac{mg}{P}$

For coffee filters dropped in groups of 1,2,3, etc., the only variable is the mass, which is proportional to the number of filters.

$m = M \leftrightarrow N$ m = total mass dropped M = mass of one filter

N - number of filters dropped

Area is constant unless the filters are deformed. Drag coefficient is constant. Viscosity is constant.

$$V_{termnlcoffeefilter} = K\sqrt{N} \quad \text{Where} \quad K = \sqrt{\frac{Mg}{P}}$$

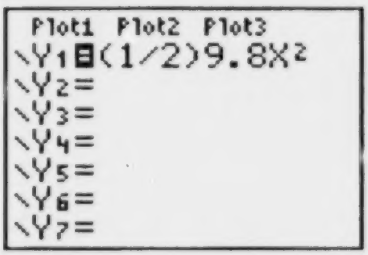
For regular 8-12 cup size, not heavy-duty industrial commercial grade, but the kind you get at K-Mart or WalMart, any brand, We have found that

$$K \cup 1 \text{ and so } V_{termnlcoffeefilter} = \sqrt{N}$$

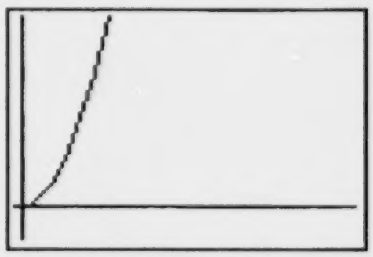
In this exercise we will attempt to verify this item.

When an object a with large mass to cross-sectional area ratio falls in air it's time distance plot is parabolic.

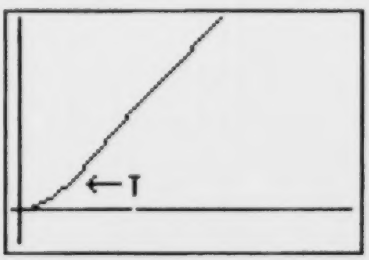
$d = \frac{1}{2}gt^2$ or on the TI 83



which with this window
(Xmin,Xmax,Ymin,Ymax)
(-0.5,1.5,-0.5,5)
gives this graph



In the case of coffee filters the curve is flattened and becomes linear like this



This complicated curve is assumed to be parabolic until terminal velocity is attained at T then becomes linear. We will, for this exercise, consider this function to be piece wise defined.

For $X = 0$ to T $Y = jX^2$ where j depends on number of filters

For $X > T$ $Y = aX + b$ where a and b depend on number of filters

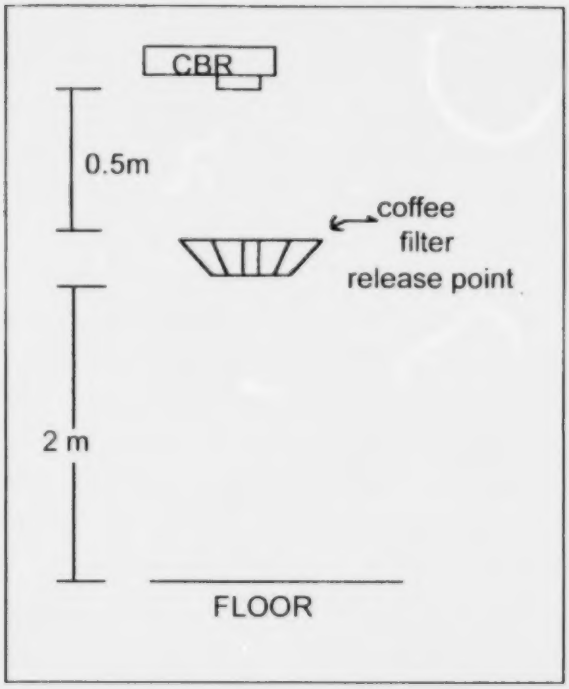
a is the slope of the time distance plot slope = $\frac{\Delta d}{\Delta t}$ = velocity

----------TERMINAL VELOCITY.

COFFEE FILTER TERMINAL VELOCITY

DATA COLLECTION
USING THE TI 83 AND THE CBR

The data collection setup should look something like this...



We have found that teams of at least three students work best for gathering the data. That's one holding the CBR at a height of about 8 foot, one running the TI-83, and one dropping the filters.

As you can see in the drawing the drop point of the filters is about half a meter below the CBR. This is because the device measures range from 0.5 to 6 meters. Drops from any closer will eliminate some of the parabolic time distance data. It should also be noted that the filter is dropped away from the CBR rather than onto it. This is for several very good reasons. The filters fall fairly straight and starting at the CBR rather than trying to land on it is considerably more productive. Additionally if they are dropped on the CBR each set of filters must be moved (if the aim was good) before the next set can be dropped. The time for each drop is somewhat less than 3 seconds so it is not difficult with this setup to get as many as six drops for each 18 second CBR activation. We really want to look at the linear part of the data and the last half meter would be lost because of being too close. We may not even attain linearity before that time for some drops.

DATA COLLECTION PROCEDURE:

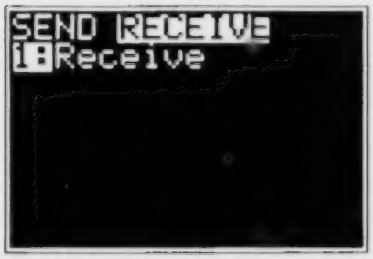
Decide how many filters will be in the drops. Count out and set-aside in at least six separate stacks the filters for the next drop. Care should be taken to insure that the same number of filters is in each stack, as they tend to stick together. Care should also be taken to not deform or flatten the filters in order to favorably compare to other drops.

Get ready to run the RANGER program with the TI83 and CBR
(if your TI 83 already has prgmRANGER skip to ***)

Connect the two units (CBR and TI-83) with the long link cable

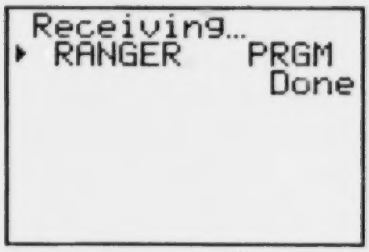
Turn on the TI 83 press [2nd] [LINK] to display the Link menu.

Press **1** to select `1:Receive`. The message `Waiting...` is displayed and the busy indicator is on



On the CBR open the hinged portion containing the screen [82/83] press while the `Waiting...` message is still displayed.

This screen will display



***

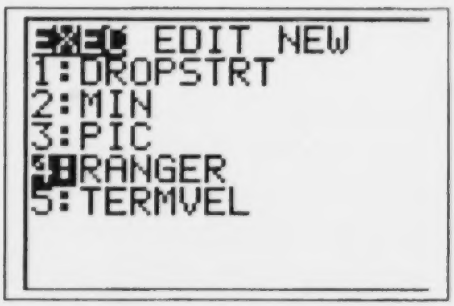
Go to the home screen (Press 2nd QUIT)

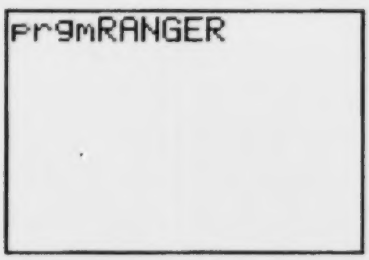
Then Press CLEAR twice

Press PRGM

This or similar screen will appear

Select #:RANGER press ENTER





This is pasted on the home screen ready for execution. Press ENTER

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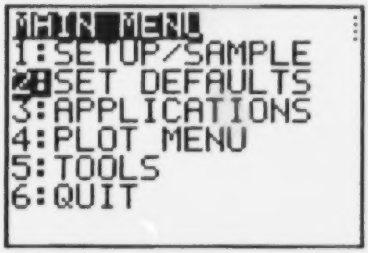
YOU WILL SEE THESE SCREENS	PLEASE FOLLOW THESE INSTRUCTIONS
	Press [ENTER]
	Select 2:SET DEFAULTS Press [ENTER]
	Press [ENTER]
	When you are ready to drop filters Press [ENTER]

Lights will flash and bells will ring (not really bells, but rather clicking) while the CBR is actuated to find Time/Distance data.

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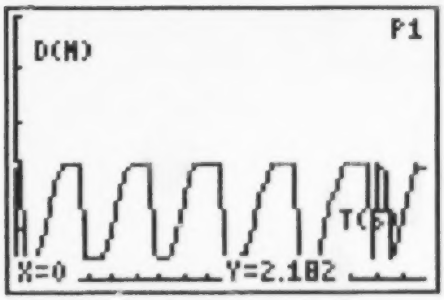
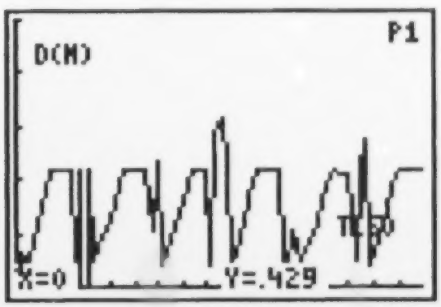
When the data collection is complete a screen like this will be displayed



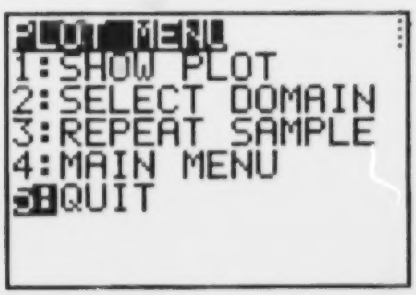
This particular screen is for a good run of six drops of 4 filters



This screen is for an OK run of six drops of 3 filters with some clutter that will not make much difference.

Look at your display and if there is not at least three drops with fairly good data you may want to try again.

Press [ENTER] This screen will appear.



To try again select
3:REPEAT SAMPLE
which will take you to the top of the previous page

When you have good data select
5:QUIT

This screen will appear

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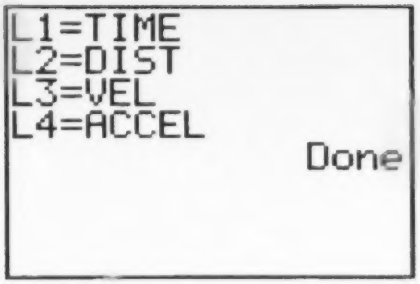


Since we did not do VELOCITY or ACCELERATION, L3 and L4 will be filled with zeros.

From the SETUP/SAMPLE screen you can choose to display VELOCITY or ACCELERATION plots. We found that for this procedure these plots were not productive.

COFFEE FILTER TERMINAL VELOCITY

ANALYZING THE COLLECTED DATA

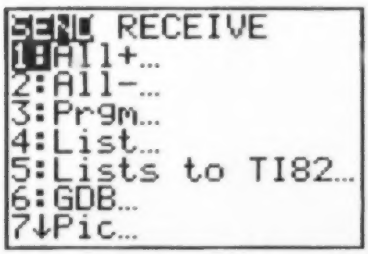
USING THE TI 83 OR TI 83 PLUS

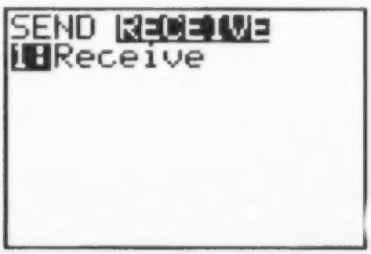
WITH MANUAL INPUTS AND MANIPULATION

Considering that data was just received by one person, either from the CBR or from the Graph Link we need to share this data with our teammates before we analyze the data. Connect the link cable to both TI 83s.

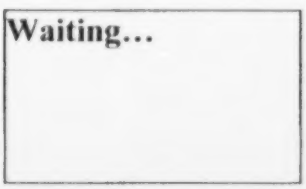
Press [2nd] [LINK]

Receiver (needs data)-- scroll over to RECEIVE





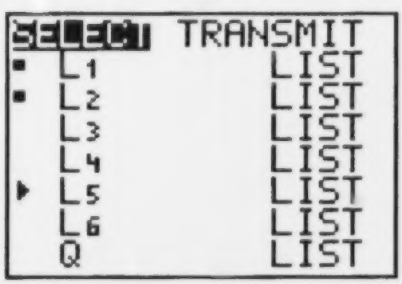
Press [ENTER]



Sender (the one that has the data)

Press [2nd] [LINK]

Select 4:List... Press [ENTER]

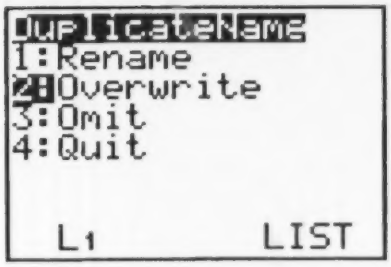


Mark L1 and L2

Scroll to TRANSMIT

Press [ENTER]

This will show on the receiving unit



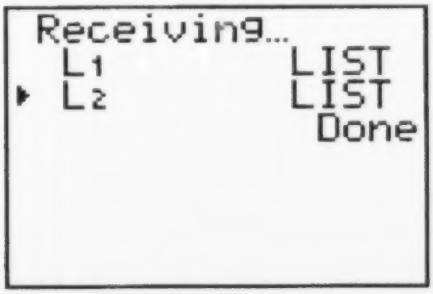
Select 2:Overwrite

Press ENTER

Then this will show

Select 2:Overwrite

Press ENTER



This process needs to be repeated
so all will have the Time/Distance data.

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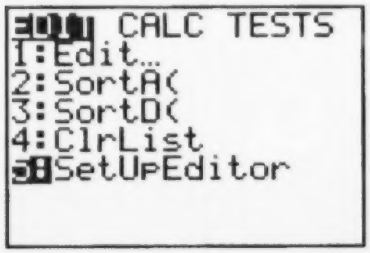
We need to save this data so that we can analyze it with out loosing it. To do this we will store the Time/Distance data lists in named lists. Of the many ways to name a list this is the way we choose to proceed.

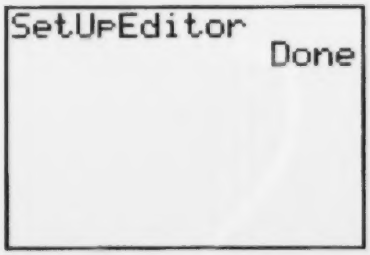
Go to the home screen (Press [2nd] [QUIT])

Then Press [CLEAR] twice

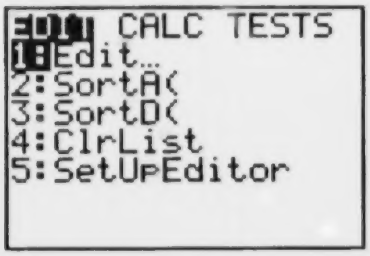
Then Press [STAT] select **5:SetUpEditor**

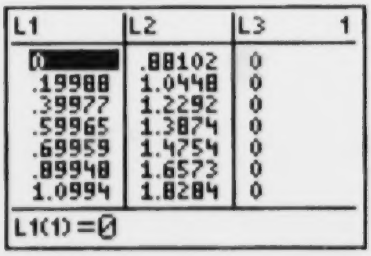


Press [ENTER] twice.



Then press [STAT] [ENTER]





L1	L2	L3 1
0	.88102	0
.19988	1.0448	0
.39977	1.2292	0
.59965	1.3874	0
.69959	1.4754	0
.89948	1.6573	0
1.0994	1.8284	0

L1(1)=0

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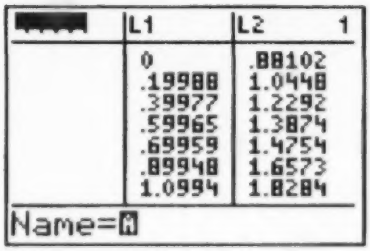
@@@

move the cursor to the very top of **L1** Press [2nd] [INS]



[redacted]	L1	L2 1
	0	.88102
	.19988	1.0448
	.39977	1.2292
	.59965	1.3874
	.69959	1.4754
	.89948	1.6573
	1.0994	1.8284

Name=[redacted]

Type the name of the new list like this

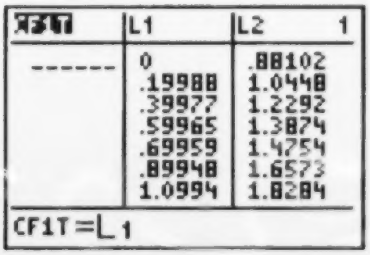
CF1T or CF1D

That's **C**offee **F**ilters # **T**ime or **D**istance

For this example we are dealing with the data for one coffee filter.

We used CF1T for the time data

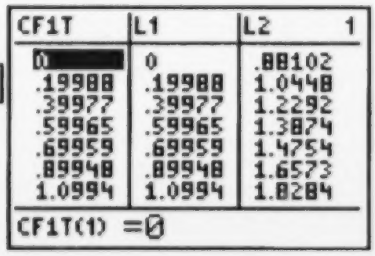
Press [ENTER] Press [2nd] [L1]



[redacted]	L1	L2 1
------	0	.88102
	.19988	1.0448
	.39977	1.2292
	.59965	1.3874
	.69959	1.4754
	.89948	1.6573
	1.0994	1.8284

CF1T=L1

Press [ENTER]

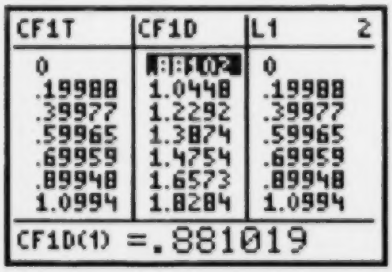


CF1T	L1	L2 1
[redacted]	0	.88102
.19988	.19988	1.0448
.39977	.39977	1.2292
.59965	.59965	1.3874
.69959	.69959	1.4754
.89948	.89948	1.6573
1.0994	1.0994	1.8284

CF1T(1) =0

L1, the time data list, has now been stored in **LCF1T**.

Go back to @@@ and repeat the process
Storing **L2**, the distance data list, into **LCF1D.** Like this.



CF1T	CF1D	L1 2
0	[redacted]	0
.19988	1.0448	.19988
.39977	1.2292	.39977
.59965	1.3874	.59965
.69959	1.4754	.69959
.89948	1.6573	.89948
1.0994	1.8284	1.0994

CF1D(1) =.881019

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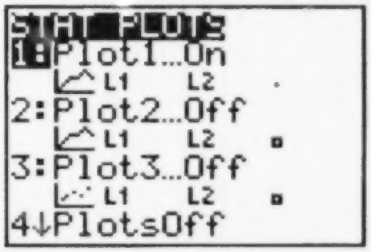
If your calculator was used with the CBR then you can view the statistical plot of the data.

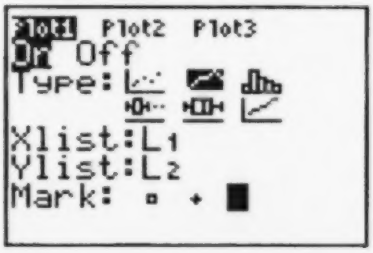
Just Press [GRAPH] and show the graph to your teammates.

Now let's all get on the same page.

Press [2nd] [STAT PLOT] Then Press [ENTER]



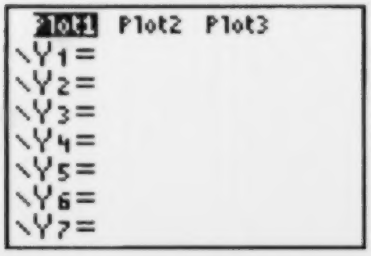
becomes



but it may not look like this. Move the cursor to highlight an item

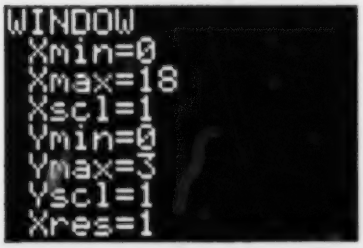
Then Press [ENTER]
highlight **On** , connected dots , make Xlist:L1 and Ylist:L2 and highlight the dot

Press [Y=]



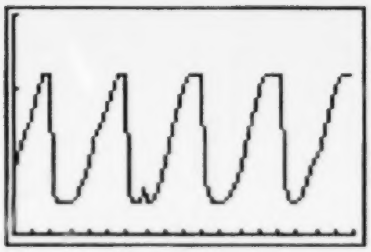
By highlighting and clearing make the screen look like this

Press [WINDOW]



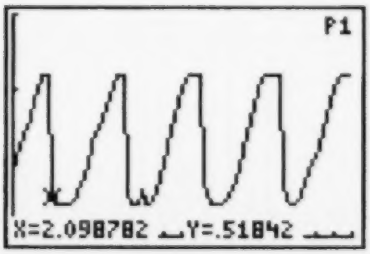
Make the screen look like this

Press GRAPH

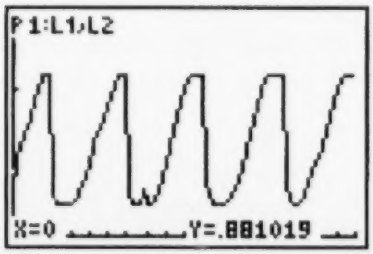


This is a Time/Distance plot of collected data. From this mentally choose a filter drop to analyze. You may get together with your team and choose different drops. Determine the start time of that drop by using the Xscale = 1sec for each mark. I will look at the second of my drops which starts at about 2 seconds.

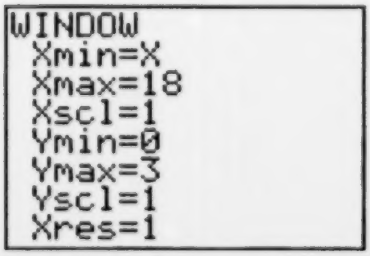
Press TRACE and scroll to the beginning of the chosen drop



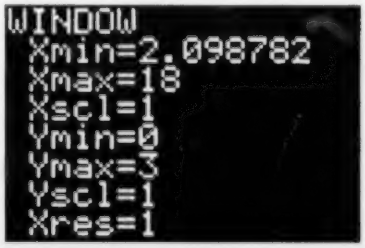
becomes

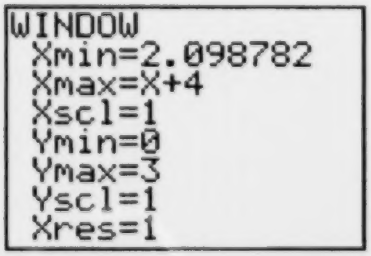


Press WINDOW and put an X for Xmin= X Press ENTER

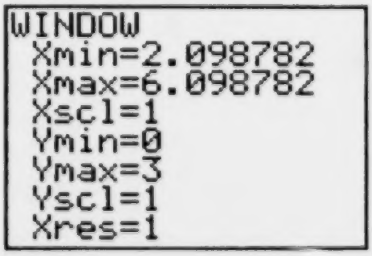


becomes with your value of X
Put Xmax=X+4
Press ENTER



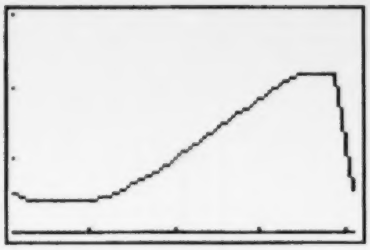


becomes with your value of X+4



Press GRAPH

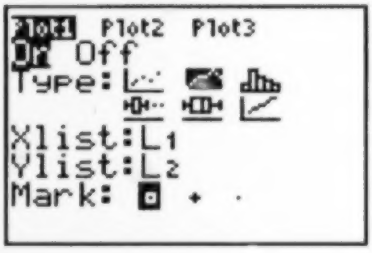
BEST COPY AVAILABLE

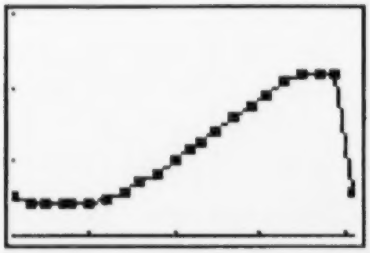


This is the Time/Distance plot for the chosen drop. It should clearly show the linear part where the filter has attained terminal velocity. To have the calculator determine the slope of this line we will select just the linear part. To see this more clearly

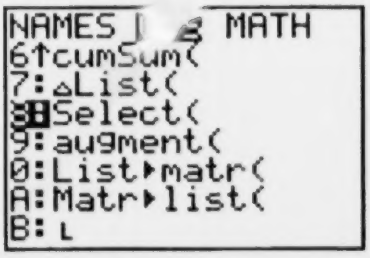
Press [2nd] [STAT PLOT] Then Press [ENTER]

select the dark block around the dot ■





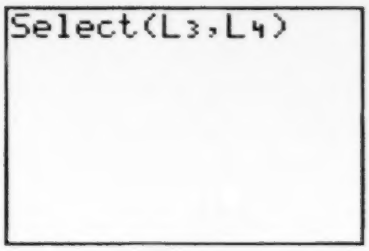
Press [2nd] [LIST] Scroll over to **OPS** down to **8:Select(**



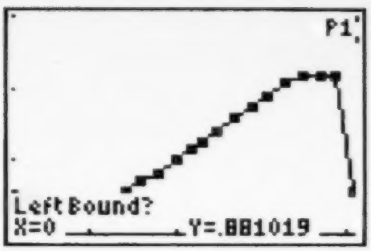
Press [ENTER]

Select(is pasted to the home screen.
Press 2nd L3 then, then 2nd L4 then)
then Press [ENTER]

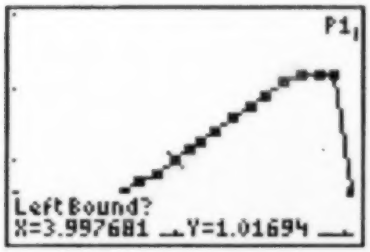
BEST COPY AVAILABLE



becomes



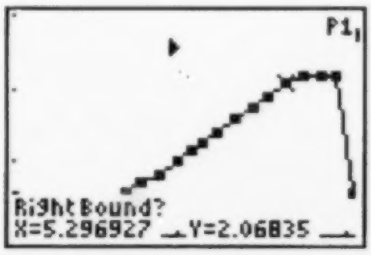
Move the marker to the right until at the start of the linear part of the graph (the left bound). Keep in mind that you are starting at the beginning of the CBR Time/Distance lists and the marker may not be in sight for some time.



Press ENTER

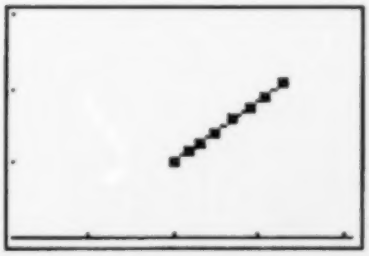
Move the marker to the right bound.

Press ENTER



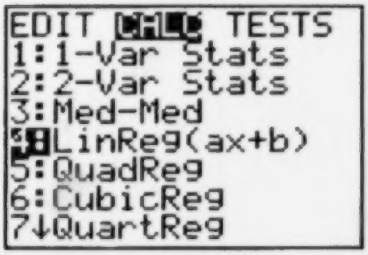
=

What is now displayed is the linear part that you selected in the same window you had. These data points are now stored in L3 and L4 .



To find the slope of this discreet data we will do a linear regression.

Press STAT scroll right to CALC and down to 4:LinReg(ax+b)



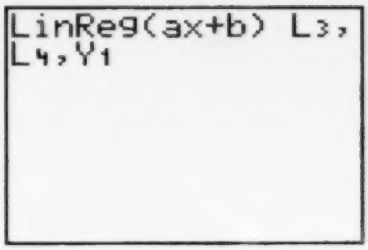
Press ENTER

LinReg(ax+b) is pasted to the home screen

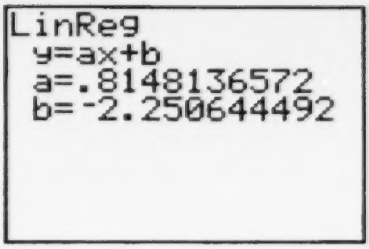
we must enter the arguments L3,L4,Y1

(Y1 is found in VARS Y-VARS 1:FunctionHighlight 1:Y1 and press ENTER to paste the home screen)

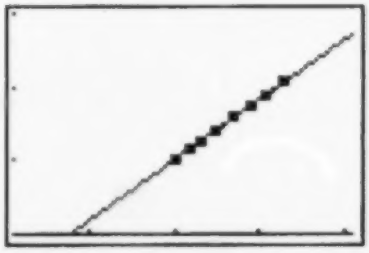
BEST COPY AVAILABLE



Press ENTER
The TI-83 will perform the regression, display it and paste the equation at Y1 = .
"a" is the slope and TERMINAL VELOCITY



for this drop
Press GRAPH



Using our data the TERMINAL VELOCITY of the second drop of one filter was .815 m/s.

COFFEE FILTER TERMINAL VELOCITY

ANALYZING THE COLLECTED DATA

USING THE TI 83 OR TI 83 PLUS

(ANALYZING ASSIST PROGRAM)

"TERMVEL"

The TI 83 Plus program was written to assist in analyzing the coffee filter time distance data. The intent is to remove the repetitious button punching necessary to analyze many coffee filter drops to provide a reasonable data base for more in-depth analysis while maintaining some uniformity in processing techniques. The program asks the user for the raw Time/Distance data lists as provided by the CBR and leads the user through a series of screens to arrive at the Terminal Velocity of a particular coffee filter drop.

The coffee filter drops and the CBR ranging pulse are non-synchronous, therefore this program uses a best guess at the shape of the nonlinear portion of the drop and from that provides only a close approximation of the distance of fall before terminal velocity is reached.

These two pieces of data are automatically stored in two TI 83 matrices for later evaluation. Matrix A and B have 13 rows for 1 through 12 filter drops and a 15 filter drop and 7 columns for six trials and an average for each number of filters dropped.. Terminal velocity is stored in Matrix A and drop distance to terminal velocity in Matrix B.

The program uses the six calculator lists L1-L6. Collected data need be stored elsewhere (see the instructions for manually analyzing data page 10).

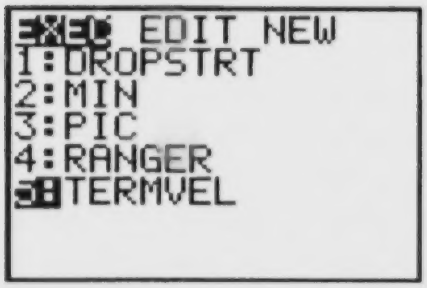
Smoothing with this program is for the specific elimination of single bits of extraneous data and not overall smoothing like the CBR does. With CBR smoothing data bits are lost and misleading information may result.

To use the program follow the instructions found on page 45 to load the program into your calculator.

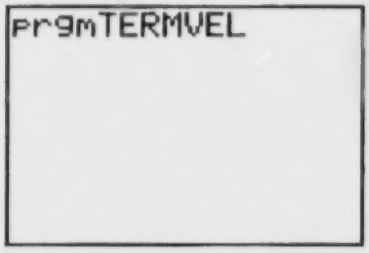
For those of you at the presentation we will link the program to you for use and retention if you want.

***Press PRGM

This or similar screen will appear



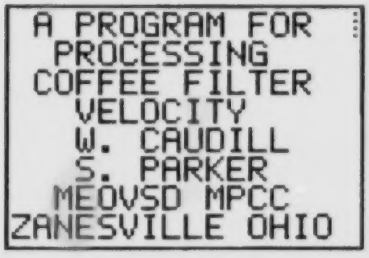
Select #:TERMVEL press ENTER



This is pasted on the home screen ready for execution. Press ENTER

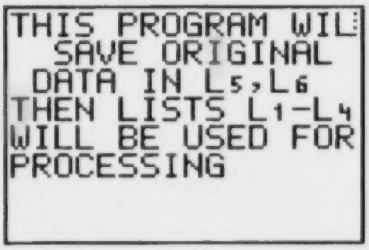
YOU WILL SEE THESE SCREENS

PLEASE FOLLOW THESE INSTRUCTIONS

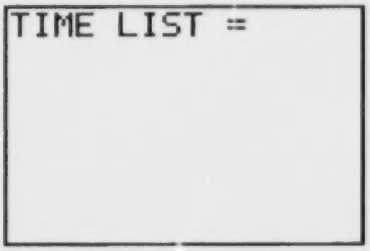


Press ENTER

BEST COPY AVAILABLE



Press ENTER



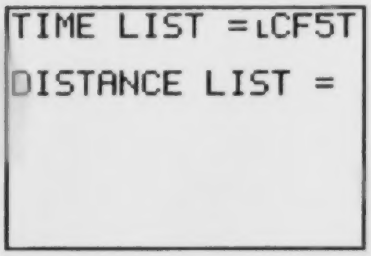
Insert the name of the list containing the time data. You can not just type it in without the list identifier character. It is best to

Press 2nd LIST

Then select the desired list.

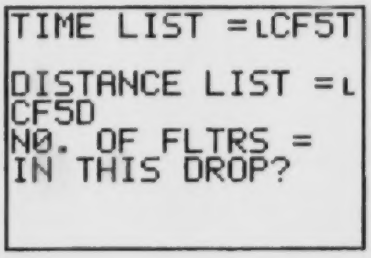
Then Press ENTER

For this demonstration we will use the data from five coffee filters dropped . These lists are stored in our LCF5T and LCF5D.



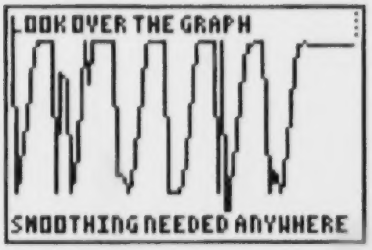
Same as above for time list.

Then Press ENTER

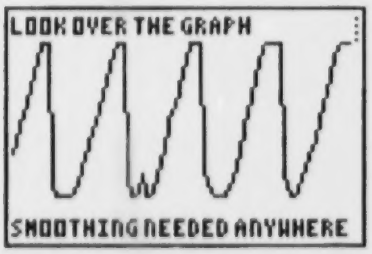


Input the number of filters dropped for the data you are using. We used 5.

Then Press ENTER



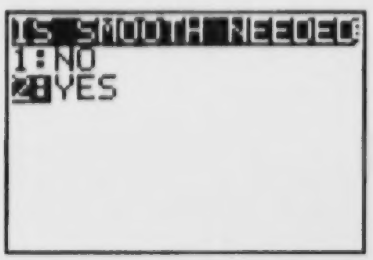
This is very much like the CBR Time/Distance plot. The question that is being asked is about inconsistent data. In this case the second drop and the fifth drop may have some "where did that come from" data.



This display is our data for one-filter drops and there appears to be no significant extraneous data.
Look over your display with this in mind.

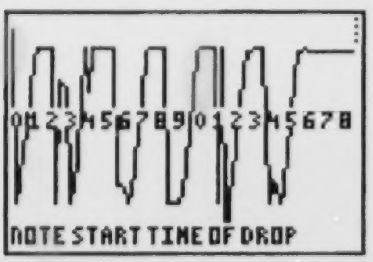
Then Press ENTER

(We returned to the five filter drop)



Let us all Press 2

to see what the smoothing is about.



The numbers across the middle of the screen are time in seconds from 0 to 18. Decide which drop to consider and note the time just a little left of its start.

_____

Then Press ENTER

```
START TIME?
```

Input the time you just noted on the previous screen. (We will look at our 2nd drop which starts at 2 seconds.)

Then Press ENTER

```
START TIME?2
DROP NUMBER?
```

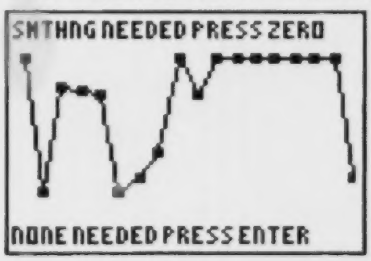
Input the number of the drop you are considering. (Ours is number 2)

Then Press ENTER

```
PLEASE WAIT
```

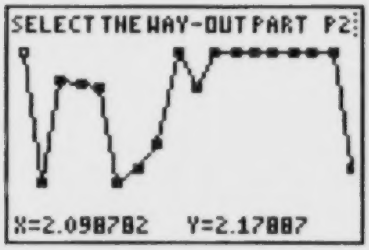
Please wait

This screen shows the extraneous data at the start of the drop and in the linear portion. The start is best guess anyway so that part in this case can be ignored. The data showing just below the last D in NEEDED ;however, will keep the linear portion of this drop from being realistic and should be eliminated. We will press 0. If no smoothing is needed (a judgment call) then



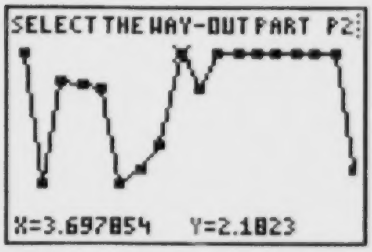
Press ENTER

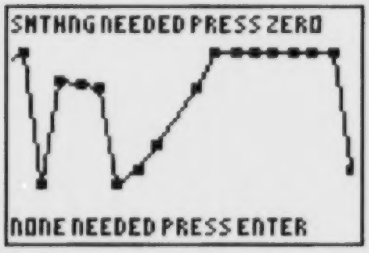
If you pressed ENTER you will skip to $$$$ if not then



Move the marker to the extraneous data.

Press ENTER



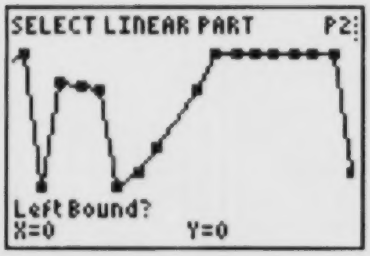


The extraneous data has been eliminated and you are asked if more smoothing is needed.

If more is needed press 0 and eliminate those points also. Otherwise

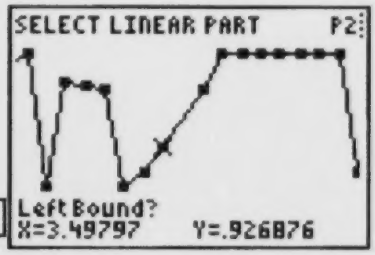
Press ENTER

$$$$



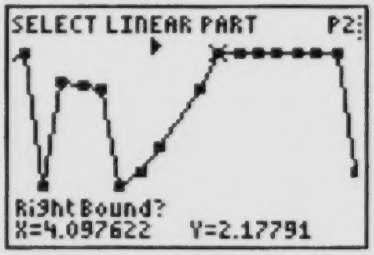
Move the maker to the left edge of the linear part and then

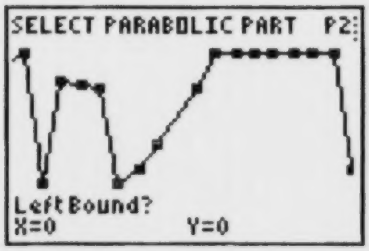
Press ENTER



Move the marker to the right end (bottom) of the linear part and

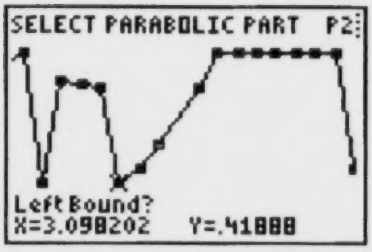
Press ENTER

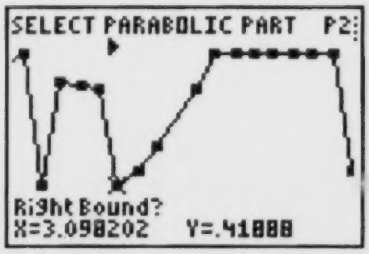




move the marker to the left (beginning) of the drop parabolic part

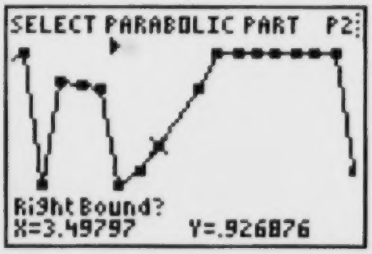
Press ENTER

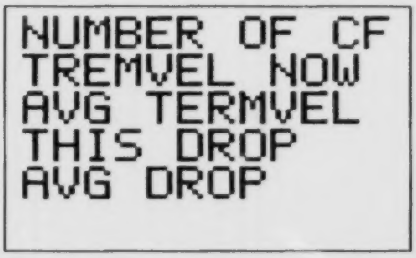




Move the marker to the end of the parabolic part (start of the linear part). You must have at least three data points.

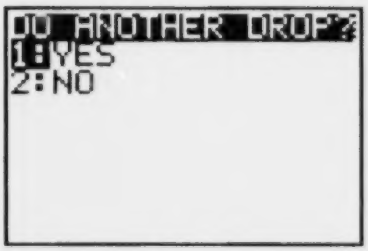
Press ENTER





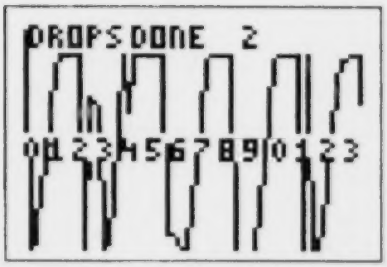
This display shows the processed terminal velocity for this drop and the average of the other drops that have been processed, as well as the distance to reach terminal velocity for this and the average of the processed drops for this number of filters

BEST COPY AVAILABLE



Press ENTER

The program returns to this screen for you to process another drop



COFFEE FILTER TERMINAL VELOCITY

COLLECTED DATA
USING THE TI 83 AND THE CBR

COFFEE FILTER CBR DATA

This file contains the data collected with a CBR at Mid-East Ohio Vocational School, Mushingum Perry Career Center, Zanesville, Ohio.

This is empirical data and will have the same inaccuracies that you would get with your students gathering data. The setup is as shown in the previous sections.

In order to use this data in a TI 83 it must be converted into a ***.8xl** file. To do this, select a single column, title and all. Open TI-Graph Link and open a File: New TI-83 List File. Paste the column onto the new list form. Select (high light) the title, cut it and past it to the "Name:" box.

The list can be imported into the TI-83 or saved for later use.

The data can be analyzed manually or by using the TI-83 Plus program TERMVEL.

NOTE: There is not enough space to put all these lists in the TI-83 at the same time unless some of it is put into archives. To process a drop both a time and a distance list are needed.

CF1T	CF1D	CF2T	CF2D	CF3T	CF3D	CF4T	CF4D
0	0.88101	0	0.42025	0	0.42931	0	2.18216
	9		7				
0.19988	1.04481	0.19988	0.41929	0.19988	0.63992	0.19988	2.18216
4		4	6	4	9	4	
0.39976	1.2292	0.39976	0.53270	0.29982	0.41888	0.39976	0.42039
8		8	2	6		8	
0.59965	1.38736	0.49971	0.60437	0.49971	0.57059	0.59965	0.41915
2					5	2	9
0.69959	1.47537	0.69959	0.82115	0.69959	0.84751	0.79953	0.42094
4		4	9	4	9	6	4
0.89947	1.65729	0.89947	1.0444	0.89947	1.15369	0.89947	0.49247
8		8		8		8	5
1.09936	1.82836	1.09936	1.27561	1.09936	1.49363	1.09936	0.71667
2		2		2		2	7
1.29924	1.99435	1.29924	1.52644	1.29924	1.84936	1.29924	1.02889
6		6		6		6	
1.49913	2.17324	1.49913	1.7836	1.49913	2.17283	1.49913	1.38036
1.69901	2.17503	1.69901	2.0045	1.69901	2.17434	1.69901	1.81518

4		4		4		4	
1.89889	2.17104	1.89889	2.18107	1.89889	2.17365	1.89889	2.1429
8		8		8		8	
2.09878	0.51842	2.09878	2.18189	2.09878	2.17379	2.09878	2.17008
2		2		2		2	
2.29866	0.41888	2.29866	2.18422	2.29866	2.17365	2.29866	2.16734
6	4	6		6		6	
2.49855	0.41737	2.49855	0.41792	2.49855	0.43961	2.49855	2.16995
	4						
2.69843	0.41970	2.69843	0.44634	2.69843	2.14057	2.69843	2.16995
4	8	4	3	4		4	
2.79837	0.41957	2.89831	1.66566	2.89831	-2.76033	2.89831	0.42025
7	1	8		8		8	
2.99826	0.41861	3.09820	0.41696	3.09820	2.2115	3.09820	0.41682
1		2		2		2	
3.19814	0.46240	3.19814	0.43302	3.29808	0.44895	3.29808	0.42080
5	7	5		6		6	7
3.39802	0.56222	3.39802	0.58789	3.49797	0.68317	3.49797	0.41764
9		9	5		7		9
3.59791	0.72477	3.59791	0.80797	3.69785	0.97054	3.59791	0.41984
3	8	3	9	4		3	6
3.79779	0.84120	3.79779	1.04907	3.89773	1.28165	3.79779	0.60244
7	4	7		8		7	8
3.99768	1.01694	3.99768	1.30609	4.09762	1.62049	3.99768	0.90532
1		1		2		1	1
4.19756	1.16124	4.19756	1.58795	4.29750	1.92844	4.19756	1.27945
5		5		6		5	
4.29750	1.25323	4.39744	1.87462	4.49739	2.16747	4.39744	1.66003
7		9				9	
4.49739	1.40384	4.59733	2.14702	4.69727	2.17461	4.59733	2.0391
1		3		4		3	
4.69727	1.58095	4.79721	2.17214	4.89715	2.16734	4.79721	2.17338
5		7		8		7	
4.89715	1.73692	4.99710	2.17969	5.09704	2.16665	4.99710	2.17777
9		1		2		1	
5.09704	1.904	5.19698	2.17969	5.29692	2.16651	5.19698	2.17695
3		5		6		5	
5.29692	2.06835	5.39686	0.41847	5.49681	2.17338	5.39686	2.17434
7		9				9	
5.49681	2.17777	5.59675	0.42300	5.69669	1.0639	5.59675	2.17434
1		3	3	4		3	
5.69669	2.18285	5.79663	0.42094	5.89657	2.34033	5.79663	0.43796
5		7	4	8		7	
5.89657	2.17777	5.99652	0.44153	6.09646	0.42231	5.99652	0.4171
9		1	8	2		1	
6.09646	0.58075	6.09646	0.52995	6.29634	0.63169	6.19640	0.41888
3		3	6	6	2	5	4
6.29634	0.41833	6.29634	0.70912	6.49623	0.88472	6.29634	0.42231

7		7	6		6	7	7
6.49623	0.41723	6.49623	0.93415	6.69611	1.17291	6.49623	0.58858
1	7	1	3	4		1	1
6.69611	0.45087	6.69611	1.19419	6.79605	1.33505	6.69611	0.87209
5	4	5		7		5	5
6.89599	0.64583	6.89599	1.45052	6.99594	1.64671	6.89599	1.17058
9	3	9		1		9	
6.99594	0.42176	7.09588	1.71783	7.19582	1.96455	7.09588	1.53029
2		3		5		3	
7.19582	0.41902	7.29576	1.99558	7.39570	2.17695	7.29576	1.89508
6	2	7		9		7	
7.39571	0.48506	7.49565	2.16967	7.59559	2.16967	7.49565	2.15965
	1	1		3		1	
7.59559	0.62674	7.69553	2.17667	7.79547	2.16514	7.69553	2.16638
4	9	5		7		5	
7.79547	0.82019	7.89541	2.16981	7.99536	2.15608	7.89541	2.16336
8	8	9		1		9	
7.99536	0.99195	8.09530	0.4208	8.19524	0.42753	8.09530	2.1639
2		3		5		3	
8.19524	1.17936	8.29518	0.42190	8.39512	2.8766	8.29518	2.16679
6		7	5	9		7	
8.29518	1.26229	8.49507	0.41929	8.59501	3.1439	8.49507	2.16706
8		1	6	3		1	
8.49507	1.45285	8.69495	0.41723	8.79489	2.24916	8.69495	0.50222
2		5	7	7		5	
8.69495	1.63848	8.79489	0.45952	8.99478	0.41806	8.89483	0.41888
6		8	4	1		9	
8.89484	1.82424	8.99478	0.66093	9.19466	0.61961	9.09472	0.41861
		2	6	5		3	
9.09472	2.01576	9.19466	0.88568	9.39454	0.92001	9.29460	0.41751
4		6	7	9		7	2
9.29460	2.16404	9.39455	1.14325	9.59443	1.28137	9.49449	0.44346
8				3		1	
9.49449	2.1672	9.59443	1.4026	9.79431	1.61184	9.59443	0.53901
2		4		7		4	8
9.69437	2.17146	9.79431	1.69765	9.99420	1.99407	9.79431	0.81003
6		8		1		8	8
9.89426	2.17557	9.99420	1.97361	10.1940	2.16555	9.99420	1.132
		2		8		2	
10.0941	0.70336	10.1940	2.16555	10.3939	2.15388	10.1940	1.52384
4		9		7		9	
10.2940	0.52927	10.3939	2.15608	10.5938	2.16665	10.3939	1.90963
3		7		5		7	
10.4939	0.43467	10.5938	2.17049	10.7937	2.16596	10.5938	2.1602
1	3	5		4		5	
10.6938	0.41902	10.7937	0.43838	10.9936	2.15937	10.7937	2.16981
	2	4		2		4	
10.7937	0.42066	10.9936	0.41861	11.1935	0.42478	10.9936	2.16912

4 9 2 2
10.9936 0.41778 11.1935 0.42012 11.3933 0.43357 11.1935 2.16953
2 6 1 9 5 1
11.1935 0.42025 11.3933 0.41847 11.5932 1.07116 11.3933 2.16967
1 7 9 3 7 9
11.3933 0.50565 11.5932 0.52542 11.8931 0.41902 11.5932 2.16967
9 5 7 5 7
11.5932 0.62592 11.6932 0.61494 11.9930 0.46597 11.7931 0.47503
7 5 2 2 4 6
11.7931 0.79287 11.8931 0.82390 12.1929 0.68592 11.9930 0.41696
6 6 5 3 3 4
11.9930 0.95200 12.0929 1.06431 12.3928 0.97026 12.1929 0.42382
4 1 8 1 3 7
12.1929 1.12513 12.2928 1.34782 12.5926 1.29496 12.3928 0.4171
3 7 9 1
12.2928 1.20984 12.4927 1.59125 12.7925 1.62955 12.5926 0.53338
7 5 8 9 9
12.4927 1.39821 12.6926 1.85609 12.9924 1.97183 12.6926 0.65256
5 4 6 4 1
12.6926 1.5723 12.8925 2.10967 13.1923 2.12889 12.8925 0.96875
4 2 5 2
12.8925 1.77069 13.0924 2.17695 13.3922 2.13864 13.0924 1.35071
2 3
13.0924 1.96071 13.2922 2.15814 13.5921 2.13384 13.2922 1.73719
9 1 9
13.2922 2.15004 13.4921 0.42204 13.792 2.13164 13.4921 2.11091
9 7 7
13.4921 2.1569 13.6920 0.42039 13.9918 2.12436 13.6920 2.14661
7 6 8 6
13.6920 2.16789 13.8919 0.42382 14.1917 0.4359 13.8919 2.14715
6 4 7 7 4
13.8919 2.17036 14.0918 0.42121 14.3916 2.19878 14.0918 2.14688
4 2 8 5 2
14.0918 2.16775 14.2917 0.44895 14.5915 2.71267 14.2917 2.14935
2 1 2 3 1
14.2917 0.69086 14.3916 0.5257 14.7914 0.41874 14.4915 2.14729
1 5 2 9
14.4915 0.4959 14.5915 0.76775 14.9913 0.61274 14.6914 2.14715
9 3 1 5 8
14.6914 0.41833 14.7914 0.98028 15.1911 0.89076 14.8913 0.42465
8 5 2 9 6
14.8913 0.41970 14.9913 1.23263 15.3910 1.18197 15.0912 2.20372
6 8 7 4
14.9913 0.42218 15.1911 1.46425 15.5909 1.51024 15.2911 1.98199
9 5 3
15.1911 0.48657 15.3910 1.71756 15.7908 1.84579 15.4910 0.41915
9 1 7 4 1
15.3910 0.59036 15.5909 1.96071 15.9907 2.12615 15.6909 0.58212

7	6	5	2				8
15.5909	0.74056	15.7908	2.14963	16.1906	2.1613	15.8907	0.89063
6	7	4	1			8	
15.7908	0.89914	15.9907	2.17461	16.3904	2.16734	16.0906	1.24197
4	2	2	9			6	
15.9907	1.07076	16.1906	2.17503	16.5903	2.17132	16.1906	1.41935
2		1	7			1	
16.1906	1.25831	16.3904	2.17599	16.7902	2.1742	16.3904	1.81888
1		9	6			9	
16.3904	1.45862	16.5903	2.17571	16.9901	2.17173	16.5903	2.16281
9		7	4			7	
16.5903	1.63257	16.7902	2.17585	17.1900	2.17173	16.7902	2.12697
8		6	3			6	
16.7902	1.81257	16.9901	2.17832	17.3899	2.17187	16.9901	2.12162
6		4	1			4	
16.9901	2.0089	17.1900	2.17859	17.5897	2.17214	17.1900	2.12148
4		3	9			3	
17.1900	2.14812	17.3899	2.17873	17.7896	2.17187	17.3899	2.12175
3		1	8			1	
17.3899	2.1845	17.5897	2.17901	17.9895	2.17228	17.5897	2.12148
1		9	6			9	
17.5898	2.16102	17.7896	2.17859	18.1894	2.17214	17.7896	2.12189
		8	5			8	
17.7896	2.17077	17.9895	2.17612	18.3893	2.17214	17.9895	2.12175
8		6	3			6	

CF5T	CF5D	CF6T	CF6D	CF7T	CF7D	CF8T	CF8D
0	2.3725	0	2.18203	0	0.42259	0	0.42451
0.19988	0.41778	0.19988	2.18189	0.19988	0.41915	0.19988	0.41764
4	6	4		4	9	4	9
0.39976	0.65681	0.39976	2.18189	0.29982	0.41888	0.29982	0.42478
8	7	8		6	4	6	8
0.59965	1.02587	0.59965	2.18189	0.49971	0.47366	0.49971	0.71914
2		2			5		9
0.69959	1.22659	0.79953	2.18175	0.69959	0.77145	0.69959	1.15492
4		6		4	8	4	
0.89947	1.6706	0.99942	2.18189	0.89947	1.14998	0.89947	1.59084
8				8		8	
1.09936	2.08949	1.19930	2.18189	1.09936	1.59399	1.09936	2.10803
2		4		2		2	
1.29924	2.17832	1.39918	2.18862	1.29924	2.07411	1.29924	2.17118

6 8 6 6
1.49913 2.18244 1.59907 2.21937 1.49913 2.17448 1.49913 2.1812
2
1.69901 2.18216 1.79895 2.30616 1.69901 2.17173 1.69901 2.17901
4 6 4 4
1.89889 2.17859 1.99884 0.4208 1.89889 2.17187 1.89889 2.1812
8 8 8
2.09878 2.17887 2.19872 0.71626 2.09878 2.17173 2.09878 0.44593
2 4 5 2 2
2.29866 0.42053 2.39860 1.10714 2.29866 2.17187 2.29866 0.42025
6 8 6 6
2.49855 1.78854 2.59849 1.57175 2.49855 0.43728 2.49855 0.42437
2 6
2.69843 1.75985 2.79837 2.04089 2.69843 0.42039 2.69843 0.51663
4 6 4 4 9
2.89831 1.71124 2.99826 2.1775 2.89831 0.42066 2.89831 0.86262
8 8 9 8 2
3.09820 0.41888 3.19814 2.18065 3.09820 0.42753 3.09820 1.37006
2 4 2 4 2
3.29808 0.61590 3.39802 2.18079 3.19814 0.54807 3.29808 1.88506
6 3 8 5 9 6
3.49797 0.92687 3.59791 2.17846 3.39802 0.87978 3.49797 2.16281
6 2 9 4
3.69785 2.1823 3.79779 2.1775 3.59791 1.36306 3.69785 2.16226
4 6 3 4
3.89773 1.67843 3.99768 0.42437 3.79779 1.8716 3.89773 2.172
8 7 8
4.09762 2.17791 4.19756 0.42053 3.99768 2.16267 4.09762 2.16047
2 4 1 2
4.29750 2.1753 4.39744 0.41655 4.19756 2.1856 4.29750 2.15745
6 8 5 6
4.49739 2.17585 4.59733 0.41874 4.39744 2.1856 4.49739 0.42025
2 7 9
4.69727 2.17557 4.79721 0.42657 4.59733 2.18532 4.69727 0.41723
4 6 3 3 4 7
4.89715 2.17571 4.89715 0.52240 4.79721 2.1856 4.89715 0.42121
8 8 5 7 8 8
5.09704 2.17928 5.09704 0.85562 4.99710 0.59077 5.09704 0.73136
2 2 1 2 8
5.29692 2.18134 5.29692 1.2929 5.19698 0.43604 5.19698 0.92193
6 6 5 5 3
5.49681 0.61562 5.49681 1.71577 5.39686 0.42039 5.39686 1.39821
9 5 9
5.69669 0.59366 5.69669 2.17022 5.59675 0.41888 5.59675 1.9548
4 4 3 4 3
5.89657 0.57389 5.89657 2.18642 5.69669 0.54450 5.79663 2.16363
8 1 8 5 9 7
6.09646 0.42094 6.09646 2.1683 5.89657 0.85671 5.99652 2.1672

2	4	2		9	8	1	
6.19640	0.53270	6.29634	2.15731	6.09646	1.30389	6.19640	2.16679
5	2	6		3		5	
6.39628	0.85191	6.49623	2.15429	6.29634	1.81655	6.39628	2.1672
9	3			7		9	
6.59617	1.24046	6.69611	2.15457	6.49623	2.16047	6.59617	2.16761
3		4		1		3	
6.79605	1.65152	6.89599	0.50675	6.69611	2.15635	6.79605	0.42053
7		8		5		7	
6.99594	2.11324	7.09588	0.42286	6.89599	2.15663	6.99594	0.41696
1		2		9		1	
7.19582	2.15979	7.29576	0.41778	7.09588	2.15731	7.19582	0.41874
5		6	6	3		5	7
7.39570	2.15841	7.49565	0.41861	7.29576	2.15704	7.39570	0.55288
9				7		9	4
7.59559	2.15828	7.59559	0.41641	7.49565	0.4197	7.59559	0.89790
3		3	3	1		3	7
7.79547	2.15828	7.79547	0.47737	7.69553	0.4171	7.79547	1.35263
7		7	2	5		7	
7.99536	2.15814	7.99536	0.78189	7.89541	0.42080	7.89541	1.5896
1		1	2	9	7	9	
8.19524	0.41778	8.19524	1.1703	8.09530	0.43371	8.09530	2.11681
5		5		3	2	3	
8.39512	0.41888	8.39512	1.61843	8.19524	0.55055	8.29518	2.15745
9		9		6		7	
8.59501	0.41861	8.59501	2.0888	8.39513	0.88445	8.49507	2.1613
3		3			2	1	
8.79489	0.41806	8.79489	2.16651	8.59501	1.39739	8.69495	2.16445
7	1	7		4		5	
8.89483	0.51993	8.99478	2.1893	8.79489	1.86776	8.89483	2.1683
9	4	1		8		9	
9.09472	0.83104	9.19466	2.15416	8.99478	2.17036	9.09472	2.17159
3	4	5		2		3	
9.29460	1.25254	9.39454	2.17777	9.19466	2.12999	9.29460	0.44661
7		9		6		7	
9.49449	1.69161	9.59443	2.17901	9.39455	2.13397	9.49449	0.42231
1		3				1	7
9.69437	2.14221	9.79431	2.1856	9.59443	2.12986	9.69437	0.41778
5		7		4		5	6
9.89425	2.16995	9.99420	0.42725	9.79431	2.12999	9.89425	0.67301
9		1		8		9	8
10.0941	2.16404	10.1940	0.42272	9.99420	0.44634	9.99420	0.84751
4		8	9	2		2	9
10.2940	2.15361	10.3939	0.45787	10.1940	0.42025	10.1940	1.28988
3		7	2	9		9	
10.4939	2.16047	10.5938	0.42904	10.3939	0.41915	10.3939	1.81463
1		5		7	9	7	
10.6938	2.15292	10.7937	0.52611	10.5938	0.48670	10.5938	2.1569

		4		5	8	5	
10.8936	0.42272	10.9936	0.87950	10.6938	0.61686	10.7937	2.17475
8		2	9		4	4	
11.0935	2.17818	11.1935	1.35153	10.8936	0.96601	10.9936	2.16802
6				8		2	
11.2934	-1.12417	11.3933	1.83371	11.0935	1.4383	11.1935	2.16734
5		9		6		1	
11.4933	0.42341	11.5932	2.1532	11.2934	1.92391	11.3933	2.16473
3		7		5		9	
11.6932	0.71626	11.7931	2.16226	11.4933	2.13837	11.5932	2.16734
2	5	6		3		7	
11.8931	1.07721	11.9930	2.16569	11.6932	2.15361	11.7931	0.41929
		4		2		6	
12.0929	1.49706	12.1929	2.15855	11.8931	2.15635	11.9930	0.41819
8		2				4	8
12.2928	1.93188	12.3928	2.16953	12.0929	2.12423	12.1929	0.42053
7		1		8		3	2
12.4927	2.1488	12.5926	2.16349	12.2928	0.49261	12.3928	0.47503
5		9		7		1	8
12.6926	2.144	12.7925	0.42753	12.4927	0.42327	12.4927	0.59077
4		8		5		5	8
12.8925	2.15265	12.9924	2.19878	12.6926	0.41737	12.6926	1.01049
2		6		4	4	4	
13.0924	2.1532	13.1923	2.19891	12.8925	0.42080	12.8925	1.44105
		4		2	7	2	
13.2922	2.15292	13.3922	0.41847	13.0924	0.52075	13.0924	1.92405
9		3			7		
13.4921	0.41957	13.5921	0.58405	13.1923	0.64761	13.2922	2.12876
7		1		5	8	9	
13.6920	0.64995	13.792	0.91671	13.3922	1.03108	13.4921	2.16418
6			6	3		7	
13.8919	0.61302	13.9918	1.34947	13.5921	1.48484	13.6920	2.15759
4		8		1		6	
14.0918	0.42300	14.1917	1.8046	13.792	1.96181	13.8919	2.15127
2	3	6				4	
14.2917	0.50977	14.3916	2.13466	13.9918	2.14413	14.0918	2.15072
1	4	5		8		2	
14.3916	0.64349	14.5915	2.15429	14.1917	2.15347	14.2917	2.15374
5	9	3		7		1	
14.5915	0.98935	14.7914	2.14345	14.3916	2.15663	14.4915	2.16445
3		2		5		9	
14.7914	1.40123	14.9913	2.14729	14.5915	2.15059	14.6914	2.16445
2				3		8	
14.9913	1.84442	15.1911	2.14674	14.7914	2.15004	14.8913	2.15086
		8		2		6	
15.1911	2.12148	15.3910	2.15031	14.9913	2.15059	15.0912	2.16075
9		7				4	
15.3910	2.11324	15.5909	2.15361	15.1911	2.15031	15.2911	2.15347

7		5		9		3	
15.5909	2.17036	15.7908	2.17901	15.3910	2.14729	15.4910	2.15072
5		4		7		1	
15.7908	2.13864	15.9907	2.18628	15.5909	2.144	15.6909	2.14976
4		2		5			
15.9907	2.13864	16.1906	0.49357	15.7908	2.13974	15.8907	2.15004
2				4		8	
16.1906	2.13837	16.3904	0.46927	15.9907	2.14317	16.0906	2.151
1		9		2		6	
16.3904	2.13837	16.5903	0.77708	16.1906	2.144	16.2905	2.15374
9		7	7	1		5	
16.5903	2.13823	16.7902	1.18856	16.3904	2.15017	16.4904	2.15072
7		6		9		3	
16.7902	2.13878	16.9901	1.6393	16.5903	2.1499	16.6903	2.15402
6		4		7		2	
16.9901	2.14139	17.1900	2.08674	16.7902	2.15017	16.8902	2.15072
4		2		6			
17.1900	2.14125	17.3899	2.13782	16.9901	2.15086	17.0900	2.15457
3		1		4		8	
17.3899	2.14166	17.5897	2.14743	17.1900	2.14743	17.2899	2.1569
1		9		3		7	
17.5897	2.1418	17.7896	2.15773	17.3899	2.14715	17.4898	2.15388
9		8		1		5	
17.7896	2.14166	17.9895	2.15182	17.5897	2.14688	17.6897	2.15361
8		6		9		4	
17.9895	2.13892	18.1894	2.15457	17.7896	2.14702	17.8896	2.15004
6		4		8		2	
18.1894	2.1385	18.3893	2.16253	17.9895	2.1499	18.0895	2.15704
5		3		6			

CF9T	CF9D	CF10T	CF10D	CF11T	CF11D	CF12T	CF12D
0	0.41861	0	2.18299	0	2.18148	0	0.42176
							8
0.19988	0.65063	0.19988	2.18285	0.19988	0.76157	0.19988	0.42877
4	8	4		4		4	
0.29982	0.83722	0.39976	2.18285	0.39976	0.54931	0.39976	0.76926
6	2	8		8		8	1
0.49971	1.30554	0.59965	2.18285	0.59965	0.50469	0.49971	1.01008
		2		2	4		
0.69959	1.86364	0.79953	0.43179	0.79953	0.51306	0.69959	1.62118
4		6		6	9	4	
0.89947	2.17022	0.99942	0.41888	0.89947	0.58748	0.89947	2.1775

8				8	3	8	
1.09936	2.16583	1.19930	0.41764	1.09936	0.86729	1.09936	2.17049
2		4	9	2		2	
1.29924	2.16651	1.39918	0.45691	1.29924	1.31734	1.29924	2.17242
6		8	5	6		6	
1.49913	2.16638	1.49913	0.59338	1.49913	1.87037	1.49913	2.17255
			6				
1.69901	2.1661	1.69901	0.99868	1.69901	2.1764	1.69901	2.17283
4		4		4		4	
1.89889	2.16665	1.89889	1.51972	1.89889	2.16926	1.89889	2.1731
8		8		8		8	
2.09878	2.17049	2.09878	2.14674	2.09878	2.16926	2.09878	2.17324
2		2		2		2	
2.29866	0.41833	2.29866	2.18505	2.29866	2.1694	2.29866	0.42314
6		6		6		6	
2.49855	0.41668	2.49855	2.17338	2.49855	2.16926	2.49855	0.41723
	8						7
2.69843	0.4171	2.69843	2.17338	2.69843	2.1694	2.69843	0.41764
4		4		4		4	9
2.89831	0.53805	2.89831	2.17338	2.89831	0.43233	2.89831	0.51512
8	7	8		8		8	8
2.99826	0.70294	3.09820	2.17338	3.09820	0.42382	2.99826	0.68427
1	8	2		2	7	1	6
3.19814	1.14943	3.29808	2.17338	3.29808	0.41737	3.19814	1.15753
5		6		6	4	5	
3.39802	1.72223	3.49797	0.42437	3.49797	0.62441	3.39802	1.68159
9					5	9	
3.59791	2.16253	3.69785	0.41696	3.59791	0.79507	3.59791	2.15553
3		4	2	3	3	3	
3.79779	2.18697	3.89773	0.42327	3.79779	1.26682	3.79779	2.18848
7		8	8	7		7	
3.99768	2.18697	4.09762	0.55617	3.99768	1.85664	3.99768	2.16336
1		2	9	1		1	
4.19756	2.18519	4.19756	0.74235	4.19756	2.17708	4.19756	2.17077
5		5	1	5		5	
4.39744	2.18724	4.39744	1.23785	4.39744	2.16926	4.39744	2.1812
9		9		9		9	
4.59733	2.18738	4.59733	1.78607	4.59733	2.16239	4.59733	0.77763
3		3		3		3	
4.79721	2.18546	4.79721	2.15031	4.79721	2.17365	4.79721	0.43014
7		7		7		7	
4.99710	0.43028	4.99710	2.15924	4.99710	2.1764	4.99710	0.41819
1		1		1		1	8
5.19698	0.4171	5.19698	2.16706	5.19698	2.17777	5.19698	0.41874
5		5		5		5	7
5.39686	0.41902	5.39686	2.17708	5.39686	0.5467	5.39686	0.47229
9	2	9		9		9	2
5.49681	0.42739	5.59675	2.16596	5.59675	0.4403	5.49681	0.64720

1	7	3		3		1	6
5.69669	0.69786	5.79663	2.16692	5.79663	0.4506	5.69669	1.05992
5	8	7		7		5	
5.89657	1.14106	5.99652	0.41957	5.99652	0.48657	5.89657	1.61363
9		1		1	1	9	
6.09646	1.6544	6.19640	0.41819	6.19640	0.78711	6.09646	2.14372
3		5	8	5		3	
6.29634	2.16185	6.39628	0.42108	6.39628	1.24856	6.29634	2.15539
7		9	1	9		7	
6.49623	2.16679	6.59617	0.51334	6.49623	1.52342	6.49623	2.15951
1		3	3	1		1	
6.69611	2.16583	6.69611	0.67054	6.69611	2.09773	6.69611	2.15539
5		5	6	5		5	
6.89599	2.16006	6.89599	1.09218	6.89599	2.15292	6.89599	2.15196
9		9		9		9	
7.09588	2.1499	7.09588	1.68145	7.09588	2.17708	7.09588	0.48396
3		3		3		3	
7.29576	2.1488	7.29576	2.18203	7.29576	2.1775	7.29576	0.41915
7		7		7		7	
7.49565	0.42492	7.49565	2.0818	7.49565	2.17708	7.49565	0.41861
1		1		1		1	
7.69553	0.41929	7.69553	2.07878	7.69553	2.17681	7.69553	0.41737
5	6	5		5		5	4
7.89541	0.41847	7.89541	2.08798	7.89541	0.4624	7.89541	0.41778
9	3	9		9		9	6
8.09530	0.42094	8.09530	2.16638	8.09530	0.42643	7.99536	0.45362
3	4	3		3		2	
8.19524	0.51636	8.29518	2.16651	8.29518	0.42574	8.19524	0.83543
6	4	7		7	9	6	8
8.39513	0.87003	8.49507	0.42396	8.49507	0.51430	8.39513	1.31927
	6	1		1	5		
8.59501	1.34975	8.69495	0.42272	8.59501	0.64418	8.59501	1.90936
4		5		4	6	4	
8.79489	1.91897	8.89483	0.42012	8.79489	1.07241	8.79489	2.1451
8		9		8		8	
8.99478	2.12148	9.09472	0.52034	8.99478	1.67692	8.99478	2.1591
2		3	5	2		2	
9.19466	2.13713	9.19466	0.71077	9.19466	2.1315	9.19466	2.15992
6		6	4	6		6	
9.39455	2.14647	9.39455	1.10769	9.39455	2.09636	9.39455	2.15649
9.59443	2.1532	9.59443	1.72113	9.59443	2.07343	9.59443	2.15992
4		4		4		4	
9.79431	2.15223	9.79431	2.11667	9.79431	2.18038	9.79431	0.45238
8		8		8		8	
9.99420	2.15704	9.99420	2.15361	9.99420	2.15237	9.99420	0.41723
2		2		2		2	7
10.1940	0.42341	10.1940	2.1488	10.1940	2.15196	10.1940	0.41929
9		9		9		9	6

10.3939 7 0.41806 10.3939 7 2.14935 10.3939 7 0.5062 10.2940 3 0.41998 3
10.5938 5 0.42423 9 10.5938 5 2.14963 10.5938 5 0.4587 10.4939 1 0.63622 2
10.7937 4 0.41902 2 10.7937 4 2.1558 10.7937 4 0.44620 6 10.6938 1.05511
10.8936 8 0.44579 4 10.9936 2 2.15594 10.9936 2 0.55370 8 10.8936 8 1.61418
11.0935 6 0.76898 7 11.1935 1 0.41833 11.0935 6 0.71475 5 11.0935 6 2.1385
11.2934 5 1.22439 11.3933 9 0.41792 11.2934 5 1.17126 11.2934 5 2.11379
11.4933 3 1.81119 11.5932 7 0.41819 8 11.4933 3 1.66923 11.4933 3 2.10885
11.6932 2 2.12999 11.7931 6 0.46666 3 11.6932 2 2.1304 11.6932 2 2.13823
11.8931 2.12134 11.9930 4 0.78793 3 11.8931 2.14523 11.8931 2.14139
12.0929 8 2.12175 12.1929 3 1.28014 12.0929 8 2.14798 12.0929 8 2.14564
12.2928 7 2.13095 12.2928 7 1.53496 12.2928 7 2.13796 12.2928 7 0.4171
12.4927 5 2.14839 12.4927 5 2.09883 12.4927 5 2.13796 12.4927 5 0.41984 6
12.6926 4 2.1234 12.6926 4 2.11352 12.6926 4 2.13809 12.6926 4 0.41806 1
12.8925 2 0.42712 12.8925 2 2.15828 12.8925 2 0.43179 12.8925 2 0.57210 6
13.0924 8 0.41819 13.0924 2.15072 13.0924 0.41888 12.9924 6 0.75347 2
13.2922 9 0.41792 3 13.2922 9 2.14757 13.2922 9 0.41915 9 13.1923 5 1.23483
13.4921 7 0.53544 8 13.4921 7 2.15127 13.4921 7 0.48739 5 13.3922 3 1.80955
13.5921 1 0.68194 2 13.6920 6 2.14715 13.5921 1 0.61768 8 13.5921 1 2.1197
13.792 1.12019 13.8919 4 0.42643 13.792 1.03767 13.792 2.11407
13.9918 8 1.66758 14.0918 2 0.42698 13.9918 8 1.59825 13.9918 8 2.16088
14.1917 7 2.13782 14.2917 1 0.44153 8 14.1917 7 2.12629 14.1917 7 2.14921
14.3916 5 2.16734 14.4915 9 0.41874 7 14.3916 5 2.05242 14.3916 5 2.14523
14.5915 3 2.15965 14.6914 8 0.66052 4 14.5915 3 2.04707 14.5915 3 2.14153
14.7914 2 2.15965 14.8913 6 1.12499 14.7914 2 2.0608 14.7914 2 2.14537

14.9913	2.15347	15.0912	1.67664	14.9913	2.06011	14.9913	2.13878
15.1911	2.15594	15.2911	2.12299	15.1911	2.04871	15.1911	2.13933
15.3910	2.15333	15.4910	2.10363	15.3910	2.05997	15.3910	2.13301
15.5909	2.15663	15.6909	2.11201	15.5909	2.06437	15.5909	2.12944
15.7908	2.15704	15.8907	2.11448	15.7908	2.06848	15.7908	2.14153
15.9907	2.15731	16.0906	2.11228	15.9907	2.0678	15.9907	2.13864
16.1906	2.15951	16.2905	2.11214	16.1906	2.0715	16.1906	2.1385
16.3904	2.16294	16.4904	2.11214	16.3904	2.0748	16.3904	2.13233
16.5903	2.16432	16.6903	2.11173	16.5903	2.0781	16.5903	2.13878
16.7902	2.16665	16.8902	2.1116	16.7902	2.06766	16.7902	2.13823
16.9901	2.1672	17.0900	2.10844	16.9901	2.06395	16.9901	2.13137
17.1900	2.16734	17.2899	2.11201	17.1900	2.07535	17.1900	2.13494
17.3899	2.1639	17.4898	2.11558	17.3899	2.0781	17.3899	2.13109
17.5897	2.1639	17.6897	2.12409	17.5897	2.0748	17.5897	2.13109
17.7896	2.15979	17.8896	2.11956	17.7896	2.0748	17.7896	2.13123
17.9895	2.15773	18.0895	2.11558	17.9895	2.07576	17.9895	2.12738

The following data was obtained with a stopwatch and not a CBR. It is for verification only.

15 Filters at from a height of 10 meters took 2.81 sec to reach the ground
15 Filters at from a height of 5 meters took 1.44 sec to reach the ground

COFFEE FILTER TERMINAL VELOCITY

TI-83 PROGRAM "TERMVEL"

This is the Coffee Filter Terminal Velocity assessment program for the TI-83 Plus or TI-83 It can be copied and pasted in the Graph Link Program to a new TI-83 Program File then linked to the TI-83 Plus or TI-83 either directly or using TI CONNECT. Download free

TI CONNECT

http://education.ti.com/us/product/accessory/connectivity/down/download.html

TI-GRAPH LINK

http://education.ti.com/us/product/accessory/connectivity/down/downgraph.html

This program reads correctly with the TI83Pluspc Font, which is loaded with the TI-83Plus GRAPH LINK Program. We have experienced some difficulty with the list name L„ , which when copied from here and pasted into TI-Graph Link comes out L" rather than L„ . These items have been highlighted and are most easily corrected before saving in TI-Graph Link.

You will need to give this a name. We used TERMVEL.
COPY AND PASTE EVERYTHING BETWEEN THESE LINES

```
ExprOff
AxesOff
Simul
FnOff
PlotsOff
ClrHome
ClrDraw
Text(1,1,"DROPS DONE"
StorePic 3
ClrDraw
Output(1,1," A PROGRAM FOR")
Output(2,1,"  PROCESSING")
Output(3,1," COFFEE FILTER")
Output(4,1,"   VELOCITY")
Output(5,1,"   W. CAUDILL")
Output(6,1,"   S. PARKER")
Output(7,1,"  MEOVSD MPCC")
Output(8,1,"ZANESVILLE OHIO")
Pause
ClrHome
Output(1,1,"THIS PROGRAM WILL")
Output(2,1,"  SAVE ORIGINAL")
Output(3,1," DATA IN L…,L†")
```

```
Output(4,1,"THEN LISTS L_-L„")
Output(5,1,"WILL BE USED FOR")
Output(6,1,"PROCESSING")
Pause
ClrHome
Input "TIME LIST =",L…
Input "DISTANCE LIST =",L†
Output(6,1,"IN THIS DROP?")
Input "N0. OF FLTRS =",R
ClrList L_,L,,Lƒ,L„
ú.05üXmin
18üXmax
ú.05üYmin
2.5üYmax
PlotsOff
FnOff
{13,7}üdim([A])
{13,7}üdim([B])
ClrHome
Plot2(xyLine,L…,L†,Ò)
Text(0,0,"LOOK OVER THE GRAPH")
Text(57,0,"SMOOTHING NEEDED ANYWHERE?")
Pause
ClrHome
0üS
Menu("IS SMOOTH NEEDED","NO",6,"YES",5
Lbl 5
1üS
Lbl 6
1+SüS
Lbl 7
ClrHome
PlotsOff
FnOff
ú.05üXmin
18üXmax
ú.05üYmin
2.5üYmax
Plot2(xyLine,L…,L†,Ò)
PlotsOn 2
For(X,0,9)
Text(25,5X,X)
```

```
End
For(X,0,8)
Text(25,5X+51,X)
End
Text(57,0,"NOTE START TIME OF DROP")
RecallPic 3
For(Z,1,6)
If [A](R,Z)>0
Text(1,35+5Z,Z)
End
StorePic 3
Pause
ClrHome
Input ("DROP NUMBER?",C)
Input ("START TIME?",T)
ClrHome
Output(3,1,"PLEASE WAIT")
TüXmin
T+3.5üXmax
ú.25üYmin
2.7üYmax
L…üL_
L†üL,
SortD(L_,L,)
1üP
While T<L_(P)
L_(P)üLƒ(P)
L,(P)üL„(P)
P+1üP
End
SortA(Lƒ,L„)
If S=1
Goto 2
Lbl 1
ClrHome
Plot2(xyLine,Lƒ,L„,_)
PlotsOn 2
Text(0,0,"SMTHNG NEEDED PRESS ZERO")
Text(57,0,"NONE NEEDED PRESS ENTER")
0üK
While K=0
getKeyüK
```

```
End
If K=102
Goto 3
If K=105
Goto 2
Goto 1
Lbl 3
Plot2(xyLine,Lƒ,L„,_)
Text(0,0,"SELECT THE WAY-OUT PART")
Trace
For(E,1,100)
If Lƒ(E)=X
Then
0üLƒ(E)
0üL„(E)
100üE
End
End
SortA(Lƒ,L„)
Goto 1
Lbl 2
ClrHome
Plot2(xyLine,Lƒ,L„,_)
Text(0,0,"SELECT  LINEAR  PART")
Select(L_,L,)
LinReg(ax+b) L_,L,,Y_
aü[A](R,C)
ClrHome
FnOff
Plot2(xyLine,Lƒ,L„,_)
Text(0,0,"SELECT  PARABOLIC  PART")
Select(L_,L,)
PlotsOff
QuadReg L_,L,
ClrList áMIN
For(J,1,40)
L_(2)-.01*JüX
aXÜ+bX+cüáMIN(J)
End
min(áMIN)üL
([A](R,C)-b)Ü/(4*a)+b([A](R,C)-b)/(2*a)+c-LüD
Dü[B](R,C)
```

```
0üW
0üG
0üH
0üM
For(V,1,6)
If ([A](R,V)>0)
Then
W+[A](R,V)üW
H+1üH
End
If ([B](R,V)>0)
Then
M+[B](R,V)üM
G+1üG
End
End
W/Hü[A](R,7)
M/Gü[B](R,7)
ClrHome
Float
Disp R
Fix 2
Disp [A](R,C),[A](R,7),[B](R,C),[B](R,7)
Output(1,1,"NUMBER OF CF")
Output(2,1,"TREMVEL NOW")
Output(3,1,"AVG TERMVEL")
Output(4,1,"THIS DROP")
Output(5,1,"AVG DROP")
Output(7,1,"PRESS ENTER")
Float
Pause
ClrHome
FnOff
Menu("DO ANOTHER DROP?","YES",7,"NO",8
Lbl 8
AxesOn
ExprOn
Stop
```

COFFEE FILTER TERMINAL VELOCITY

CONCLUSION

55

We have analyzed the collected data by hand and by use of the assisting program and should now have enough information to draw some reasonable conclusions. We will fill out this table with our collected data.

Number of Filters L1				Average L2
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
15				3.6496

Now we will put this data in a list and display it to see if some conclusion can be reached,

Press [GRAPH] [ENTER]

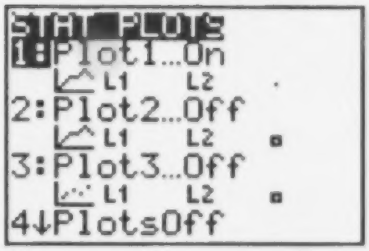
Scroll to the top of L1 Press [CLEAR] [ENTER]

Then enter the number of filters that go with our data above.

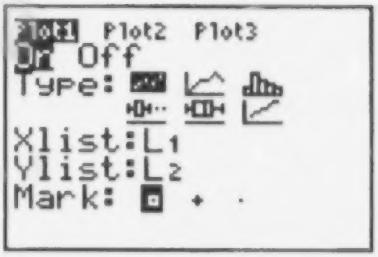
Scroll to the top of L2 Press [CLEAR] [ENTER]

Then enter the terminal velocity of the filters from our data above keeping the number of filters in L1 lined up with the data in L2.

Press [2nd] [STAT PLOT] Then Press [ENTER]



becomes

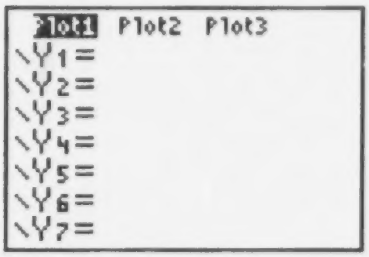


but it may not look like this. Move the cursor to highlight an item

Then Press [ENTER]
highlight **On** , not connected dots , make **Xlist:L1** and **Ylist:L2**

and highlight the box dot

Press [Y=]



By highlighting and clearing
make the screen look like this

Press [ZOOM] [9]

If we have enough data we may get something like this. We said in the beginning that the terminal velocity was proportional to the square root of the number of filters and hoped to show that.

$$V_{termnlcoffeefilter} = K\sqrt{N} \quad \text{Where} \quad K = \sqrt{\frac{Mg}{P}}$$

For regular 8-12 cup size, not heavy-duty industrial commercial grade, but the kind you get at K-Mart or WalMart, any brand. We have found that

$$K \cup 1 \text{ and so } V_{termnlcoffeefilter} = \sqrt{N}$$

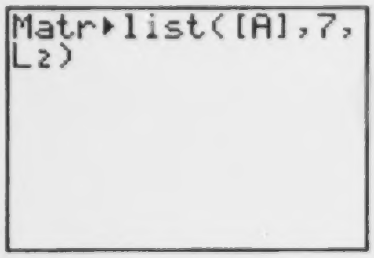
In this exercise have attempted to verify this item.

The data for 15 coffee filter terminal velocity came from use of our school's Power Lineman Program's bucket truck. We dropped 15 filters from 10 meters and timed the drop with a stopwatch. Then dropped from 5 meters and timed the drop. This gave us the time for the first 5 meters for both drops and left the time for the last 5 meters of the 10 meter drop at terminal velocity..
When many drops have been processed the above graph is even more convincing. Data from the seventh column of matrix A can be transfered to a list with built-in commands.

Press [2nd] MATRIX

Scroll over to Math Down to 8:Matr>list Press [ENTER]

This is pasted to the home screen and with these arguments will put the processed data in the right place.



Press [ENTER]

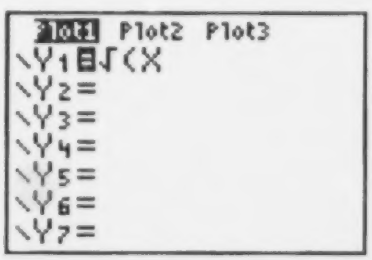
To complete the transfer

Press STAT ENTER

Scroll to the top of L1 Press CLEAR ENTER

Then enter the number of filters that go with non-zero data.
Delete entries that are zero in both L1 and L2.
Add the data from the 15 filter drop (3.6496)

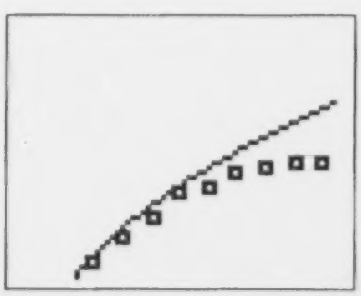
Press Y=



Enter

$$Y_1 = \sqrt{X}$$

Press ZOOM 9



Actually our data looks more like K= .9 with more deviation at higher numbers of filters.

The data from many drops provided a similar plot for distance of fall before reaching terminal velocity. It turned out to be inconclusive for us. Thyis is an area your student may find interesting. The use of different collection time periods in the CBR may be somewhat productive for refining this data.

The use of the velocity and acceleration plots with the CBR was not at all worthwhile for us but leaves other good material for student investigation.

Teach well.

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